13 Do we need a number sense?

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> From year to year, species after species, ever more specialised mental organs have blossomed within the brain to better process the enormous flux of sensory information received, and to adapt the organism's reactions to competitive or even hostile environments. One of the brain's specialised mental organs is a primitive number processor that prefigures, without quite matching it, the arithmetic that is taught in our schools . . . This "number sense" provides animals and humans alike with a direct intuition of what numbers mean.
>
> (Dehaene, 1997, pp. 4–5)

> I began this article by asking how we come to have knowledge of number. I hope to have provided the answer – we are built specifically to do so.
>
> (Wynn, 1998, p. 302)

In its simplest form, the question of domain specificity asks only: When people process information, do they use specific processes for specific tasks, or do they use general purpose processes for many different tasks? For those who study adult cognition, this question is relatively straightforward. But for those who study cognitive development, domain specificity has taken on special meaning because it has been invoked to explain not only how information is processed, but also how concepts originate and how learning takes place. Domain specificity is often linked with nativism, leading to the proposal that human infants have instinctive or core knowledge for certain domains that gives us a leg up in learning (Chomsky, 1980; Dehaene, 1997; Fodor, 1983; Gelman, 1990; Leslie, 1994; Spelke & Tsivkin, 2001; Wynn, 1998). In this chapter, we evaluate the evidence regarding one such domain: number. Do humans need a number sense to learn the meanings of small numbers, or can domain general processes better explain what we already know about this process?

Do we need special help for number learning?

Domain specific accounts of numerical development often begin with the assertion that number learning would be difficult, if not impossible, without

the help of a domain specific mechanism (Gelman, 1991; Spelke & Tsivkin, 2001). As Spelke and Tsivkin put it, "Number is arguably our most abstract system of knowledge . . . How can children ever come to understand counting if they do not already understand the entities that counting singles out?" (p. 84).

This assertion has merit because the property of numerosity, though omnipresent, is not particularly obvious. Consider, for example, what is required to learn the meaning of "three." As Quine (1960) pointed out, the referents for concrete nouns, like "rabbit," are remarkably indeterminate on closer analysis. But for attributes like threeness, the indeterminacy problem is even more profound. Because number is a property of sets, there is not an object to point toward. Instead, the boundaries of a collection must be established before it, as an entity, can act as a referent for a count word. There also is extraneous information to ignore. To learn what "three" means, children must ignore the properties of each individual object in the collection, just as they must ignore the specific properties of a rabbit, such as soft and furry, to learn the word "rabbit." However, they must also ignore properties of the collection as a whole, such as its total length, density, or area.

Beyond isolating the property of number, young learners also must recognize numerical equivalence classes for each set size and learn what to call these groups. Forming equivalence classes requires the realization that diverse individuals with a range of distinctive features are somehow alike. So, just as children come to see many breeds of rabbit as similar, they also must see many collections of three as similar. However, unlike rabbits, collections of three may bear little, if any, resemblance to one another. Consider the commonalities between three planets and three jumps. To learn the names for these equivalence classes, children must map the words in their native language to the abstract ideas to which they refer. But, unlike the names for nouns and other attributes, children learn the count words as part of the counting sequence, as well as learning them as labels for different set sizes. Thus, to understand the meaning of "three," they must sort out both these and many other meanings and usages of numerals and number words (see Mix, Sandhofer & Baroody, 2005, for further discussion).

Clearly, number learning has unique challenges. But does this mean that it requires unique processes? Some have argued that without number-specific representations built in, children would be unable to surmount the complexity of the number learning problem.

Domain specific models of number development

Over the past 20 years, two models have emerged as the leading domain specific explanations for early numerical development: Skeletal Principles and Core Knowledge.

Skeletal principles

On this view, number learning is guided by counting principles that are embodied in an innate representation for number called the accumulator mechanism (Gallistel & Gelman, 1992; Gelman, 1991). This mechanism works by emitting pulses of energy at a constant rate. When an item is counted, a gate opens that passes energy into a storage unit (i.e., an accumulator). Although there is not a one-to-one relation between pulses and items, the amount of energy per item is roughly equal. Thus, the resulting fullness of an accumulator represents the cardinality of the set. It has been argued that accumulators are used to remember set sizes, compare one set to another (e.g., 3 > 2), and solve calculation problems (e.g., 1 + 2 = ?) (e.g., Wynn, 1995, 1998; Gelman, 1991). The accumulator representation has no upper limit. However, as magnitude increases, the variability, or noise, around the exact numerosities increases. Therefore, in accordance with Weber's law, discriminability decreases as either the set sizes increase, the difference between set sizes decreases, or both.

A key claim is that the accumulator representation operates according to the same principles as verbal counting (Gallistel & Gelman, 1992; Gelman, 1991). For example, to count a set of pebbles verbally, one tags each pebble once and only once with a count word (one-to-one principle). If a pebble is omitted, or is tagged more than once, the last count word will not represent the cardinality of the set. According to the skeletal principles view, the accumulator also obeys the one-to-one principle because energy is gated into the accumulator once and only once for each item. It has been argued that the accumulator obeys all five counting principles, including this and stable order, cardinality, abstraction, and order irrelevance (Gallistel & Gelman, 1992). In fact, these investigators saw so many parallels between the accumulator and verbal counting that they called enumeration via accumulator "preverbal counting" (Gallistel & Gelman, 1992). Still, whereas skeletal principles are thought to provide an outline for developing a concept of number, experience with objects and verbal counting is needed to flesh this framework out. Thus, the idea of skeletal principles is rather like a language acquisition device for number – nature provides the conceptual slots, but experience is required to fill them (Gelman, 1993, 1998).

Clearly, an inborn counting mechanism would go a long way toward surmounting the challenges of number learning. First, by directing attention toward discrete number, this mechanism would solve a large part of the indeterminacy problem. As Gelman (1998) put it, "first principles [contribute] by focusing attention on inputs that are relevant for acquisition of concepts and providing a way to store incoming data in a coherent fashion" (p. 562). On this view, competition from other percepts would be diminished because domain specific structures give number information privileged status and salience. Second, by providing an amodal representation of cardinal number, the accumulator would help children see disparate

sets as equivalent. Though various collections may differ in many respects, these differences would be stripped away once the collections were represented as featureless magnitudes. This abstraction not only would support the development of number categories, but could also facilitate the mapping of number words by providing unambiguous referents. Children, who are predisposed to isolate number from the perceptual stream, should also be more likely to map number words to referents correctly. And as an additional benefit, children should have less trouble acquiring conventional counting skills because the accumulator follows all the principles of verbal counting. If the rules that govern counting are familiar, then learning to count would boil down to implementing known rules with language-specific terminology – a far cry from deducing the rules while also learning the words.

Core knowledge

Similar developmental benefits are provided in a second domain specific learning account that endows infants with core knowledge for number (Spelke, 2003; Spelke & Tsivkin, 2001; Wynn, 1992a, 1995, 1998). In Spelke's version, the proposed core consists of two distinct systems for representing number. One system uses a tracking mechanism that assigns a mental token to each object in a group. These tokens function as pointers to the objects' locations. Because there is a one-to-one relation between tokens and objects, the set of tokens can be used to represent the exact number of objects. However, only a few pointers can be active at any one time due to constraints on selective attention. Furthermore, although the representation preserves the individuality of the objects, it does not provide a representation of the whole group (i.e., in the way that a number word like ''three'' verbally represents a set's cardinality).

The other system (also identified as the sole core knowledge structure by Wynn, 1995, 1998) represents large sets, but only approximately. It is based on the accumulator mechanism described above. However, proponents of the core knowledge view do not emphasize the accumulator's parallels to verbal counting. In fact, Wynn (1995, 1998) has argued that the lack of functional parallels between counting and the accumulator is what makes learning to count so difficult. Instead, these investigators focus on the strengths and limitations of the accumulator – in Spelke's case, with relation to the object tracking representation. In this regard, she notes that the accumulator is inherently inexact, even for small sets, because there is not a one-to-one relation between pulses and items (though there is a one-to-one relation between gate openings and items). Also, in contrast to the exact system, this representation does not preserve the individuality of the items, though it does represent the group as a whole.

Thus, Spelke's core knowledge account holds that both systems have inherent limitations – the first being limited to set sizes that the object

tracking mechanism can handle (i.e., <4) and the other being limited to rough estimates. Furthermore, though both systems represent an aspect of number, they do not interact so as to provide the basis for a complete number concept (i.e., the ability to represent a collection composed of individual items). Only verbal counting, she (Spelke, 2003; Spelke & Tsivkin, 2001) has asserted, allows people to represent collections composed of individuals, and do so for all set sizes exactly.

Despite their limitations, it is argued that these core knowledge systems constitute the conceptual foundation for subsequent learning and, therefore, play a crucial role in numerical development. They solve many of the same problems that preverbal counting solves in the Skeletal Principles view. Like preverbal counting, core knowledge serves to direct attention toward discrete number, thus making numerical interpretations of the count words more likely. Like preverbal counting, core knowledge provides amodal representations of cardinal number, thereby supporting the formation of numerical equivalence categories. The main difference is that in the core knowledge view, the rules of conventional counting are not innately available.

In summary, though these views vary in their treatment of certain points, they share one key assumption – that children use number-specific knowledge systems to acquire more mature concepts and conventional skills. In each account, the problems of number learning are reduced by endowing children with knowledge structures that direct attention toward number, support comparisons and abstraction, and provide an organizing framework for mapping words to meaning. Though these domain specific systems are incomplete in and of themselves, they are thought to reduce enough variability to make number learning tractable.

Plan for the chapter

In this chapter, we aim to evaluate whether number learning actually is supported and guided by domain specific processes – by an innate number sense. This is challenging because, whereas it may be possible to show that processes exist, it is nearly impossible to prove they do not. To draw an extreme analogy, one could propose that number concepts develop after the number fairy sprinkles magic dust on children in their sleep. Though you could interpret a lot of behavioral evidence in terms of this explanation, it would be impossible to falsify. For example, if you videotaped a sample of children every night and never saw the number fairy, a fairy-theorist could argue that she was there, but she was too small to be seen by the naked eye.

A further complication is that domain specific learning and domain general learning are not mutually exclusive. In fact, there is widespread agreement that domain general processes are involved in all learning, including number learning. The argument of domain specific theorists is that these processes are not sufficient on their own. Thus, simply showing

that number learning involves domain general processing cannot rule out domain specific processing.

How, then, can the question of domain specificity be addressed? In the present chapter we take three approaches. First, we ask whether particular domain general mechanisms can solve particular number learning problems. This is different from simply claiming that domain general processes are involved in number learning. At a general level, any mental activity requires domain general components, such as attention, perception, or memory. But when we consider domain general mechanisms that are more detailed, they often are either distinguishable from, or redundant with, the proposed domain specific mechanisms.

Second, we will look for behavioral signatures of these domain general processes that have been documented in other concept learning. Because the explanations we entertain are detailed, they produce idiosyncratic patterns of learning. These patterns have been reported for learning a variety of noun and adjective categories (e.g., color) as well as analogical reasoning and conceptual mapping tasks. If the same signatures were observed in number development, it would be strong evidence that the same mechanisms are involved.

Third, we consider whether there is any additional evidence that compels a domain specific account. Even if the basic problems of number learning can be explained with domain general processes, it is possible that some behavioural evidence cannot. Indeed, the idea of domain specific number learning arose largely in response to evidence for numerical sensitivity in infants. We consider both the validity of this evidence and whether it requires a domain specific explanation.

A domain general account of number development

Though number development involves many conceptual components, we will focus on three achievements in particular: (1) isolating number from the perceptual stream; (2) forming small number categories; and (3) bringing meaning to number words. These accomplishments likely involve both verbal and nonverbal processing. As such, they not only emerge in the age range most likely to benefit from either skeletal principles or core knowledge, but also form the foundation for a range of other skills and ideas. We have already discussed the particular challenges inherent to these developments in the realm of number, and we have seen how prominent domain specific accounts can explain children's achievement of them. Now, let us consider whether domain general mechanisms provide a plausible alternative.

How do children isolate number from the perceptual stream?

Number applies only to collections of individuals. Thus, to have a notion of number, one must maintain and coordinate two interpretations of reality:

(1) there are individual objects and (2) some of these objects form a coherent whole – a collection. To explain the origins of these interpretations, domain specific theorists build them into the baby. For example, Spelke (2003; Spelke & Tsivkin, 2001) contends that object tracking yields a representation of individuals, and the accumulator yields a representation of collections as wholes. Though she has argued that the coordination of these notions requires language, she maintains that the ideas are supplied by evolution.

Setting aside, for the moment, the claim that these knowledge systems are innate, we should point out that neither object tracking nor the accumulator is a specialized mechanism for number. Object tracking is a perceptual mechanism for representing objects and their locations (Kahneman, Treisman, & Gibbs, 1992; Pylyshyn, 1989). The ability to parse a scene into discrete objects and track these through space serves many functions, including navigation, object representation, object identification/naming, and object manipulation. Indeed, object individuation is so fundamental to human cognition that it is hard to imagine how most other processes could operate without it. Though the link between numerical cognition and object representation is obvious (i.e., it would be impossible to perceive number without individuation), that does not mean that the processes underlying individuation were evolved to enable humans to think about number (Scholl & Leslie, 1999). Instead, object tracking may be the quintessential domain general process.

Similarly, the accumulator could apply to a variety of mental activities. There is evidence from rats that it underlies the estimation of time (Meck & Church, 1983). It could also, in principle, support estimates of intensity, size, and spatial extent. In fact, one could argue that the accumulator is better suited to continuous applications such as these because they do not require the effortful and potentially error-prone step of gating energy in segments (Mix, Huttenlocher, & Levine, 2002a). From this perspective, it seems more likely that the accumulator was evolved for non-numerical uses and was perhaps coopted for numerical processing, rather than the other way around. Of course, there are other ways children could come to see collections as coherent wholes besides representing them with an accumulator. We return to this issue later. For now, we wish only to acknowledge that this process does not constitute a domain specific endowment, even if it turns out to underlie numerical development.

So, one possible answer to the question of how children isolate number without domain specific processes is that they rely on processes that are inborn, but domain general. However, this redescription still builds quite a lot into the baby unnecessarily, given recent discoveries about very early learning. Multiple studies have demonstrated that infants readily extract statistically reliable patterns from a variety of perceptual data, including auditory sequences (Saffron, Aslin, & Newport, 1996) and visual scenes (Kirkham, Slemmer, & Johnson, 2002), after even brief exposures.

Moreover, infants recognize these patterns in subsequent, unfamiliar situations (Gomez & Gerken, 1999). This means that the conceptual precursors to number (i.e., individuation and colligation) could develop rapidly over the first year of life, rather than being innate.

Individuation via statistical learning

Infants are bombarded with information about the physical world starting at least as early as birth (and perhaps earlier if we include encounters with one's own hands and feet). The number of distinct objects infants observe and contact in a single day at home far outstrips the number of stimuli presented in a statistical learning experiment. There can be no doubt that everyday experience provides enough data about objects to support the extraction of statistically reliable patterns. And according to the literature on object concepts, not only do such patterns exist, but infants respond to them in a predictable sequence that suggests gradual abstraction over time.

Adults use many cues to parse the environment into separate objects, including colour, texture, and shape. But perhaps the most reliable test is whether all the parts move together or separately. Indeed, even adults overlook dramatic changes in an object's surface features (e.g., one person changing into another), as long as the object occupies the same position or trajectory in space (Scholl, Pylyshyn, & Feldman, 2001; Simons & Levin, 1997). This use of movement and space may reflect an innate bias, but it also could be learned. A newborn baby, who lacks the strength to even sit up, nonetheless observes other animates, most notably people and household pets, in nearly constant motion. This movement provides exceptionally reliable cues that Mom, for example, is not part of the wall, the table, or the bed. The limitations of newborns' visual systems actually may serve to increase their sensitivity to the patterns in movement information by reducing the salience of competing featural cues. As in other "less-is-more" accounts (e.g., Newport, 1990; Regier & Gahl, 2004), early lack of visual acuity may make infants particularly good at using movement cues because these would make up the bulk of their input, by default.

Statistical learners are sensitive to correlations among features. So, once infants see moving objects as unitary, they should be able to extract other reliable patterns based on the correlated features of these units. For example, movement may tell them that the family dog is not part of the rug or the furniture. But this moving blob also consists of several other correlated features. It always has roughly the same shape. It is covered with brown fur. It moves a certain way. In contrast, another moving blob (e.g., Mom) may be tall. It may talk and smile. Good things might happen when it picks you up. Enough exposure to these bundles of correlated features should allow statistical learners to realize that colour, shape, texture, and sound also indicate unity and distinctiveness. Eventually, these cues may be enough, in and of themselves, for infants to perceive individuality. And the

learning we have described so far is what could occur in immobile infants. Once infants begin to move around and manipulate objects, the amount of information they receive about individuality would increase exponentially. In summary, given the correlated structure of individual objects and the rapidity with which infants can learn correlated structures, it is quite plausible that the perception of individuals is learned.

Specific patterns in the object individuation literature lend support to this hypothesis. Multiple studies have described a developmental progression in the types of cues infants use to parse their visual world (Kellman & Spelke, 1983; Needham, 1999; Slater, Morrison, Somers, Mattock, Brown, & Taylor, 1990; Wilcox, 1999; Wilcox & Baillargeon, 1998; Xu & Carey, 1996; Xu, Carey, & Welch, 1999). Consistent with our account, this progression begins with the use of movement or spatio-temporal cues. In Kellman and Spelke's (1983) seminal rod and box experiments, 4-month-olds perceived two ends of a rod protruding from behind a screen as one continuous piece, as long as the two ends moved together. Subsequent research has shown that older babies also use movement cues to tell objects apart. For example, when shown a duck and a truck emerging simultaneously from behind a screen and then returning, 10-month-olds seem surprised to see only one object when the screen is lowered (i.e., they look longer at one object versus two) (Xu & Carey, 1996). Apparently, they realize that the same object cannot appear in more than one place at the same time. However, when only featural information is available (i.e., when the duck was displayed alone and then hidden behind the screen while the truck was displayed), 10-month-olds respond as if they no longer represent the objects as distinct individuals. The fact that the duck and the truck did not look at all alike was not enough information to tell babies that these were separate objects. They apparently needed to see the objects occupying different locations. Thus, movement/spatiotemporal information seems to be the fundamental cue to individuation used by infants. Still, though use of this information emerges early, it is not present from birth. When newborns were tested with the rod and box procedure, they reacted to the test displays as if they perceived two small rods, rather than one continuous piece (Slater et al., 1990). This indicates that even movement cues may be learned during the first four months of life.

Further research has mapped out the use of various featural cues in infancy (Needham, 1999; Wilcox, 1999; Wilcox & Baillargeon, 1998; Xu & Carey, 1996). Though differences in testing procedures have led to disagreement about the particular ages involved, the existing studies all show that infants begin to use different cues at different times, in the same basic progression. This starts with the use of movement and form features, including size and shape. Somewhat later, surface features are used, beginning with pattern. Relatively late in development (at 11.5 months according to Wilcox) infants begin to use color. This gradual acquisition of cues to individuation is consistent with the idea that infants learn what features go

together after massive experience with objects. From this perspective, there are at least two reasons why some cues would be noticed before others. One is, as noted above, that changes in infants' visual acuity may increase the salience of certain information (e.g., movement, size, and shape) over information that requires better vision to discern (e.g., pattern). Another reason may be that some cues are more tightly coupled with objecthood than others. For example, because many objects are multicolored, color may be a less reliable cue to individuation than shape. If so, then infants may not expect color information to indicate separate objects until they have amassed enough experience to know that it can sometimes be diagnostic.

Colligation via categorization

Next, we turn to the second conceptual precursor to number: the notion of collections as undifferentiated wholes. Recall that in both of the domain specific accounts, the accumulator supplies this notion by converting perceived collections into mental magnitudes for which the individuality of their constituents is obscured. It may be true that such a representation would accomplish this. However, it is not clear how the accumulator solves an arguably more basic problem – namely, that of perceiving the collections in the first place. In other words, there is a chicken–egg problem inherent to the domain specific argument. To enumerate a collection using the accumulator, the infant must first see a particular subset of objects as a collection. Yet, if they see a subset of objects as a collection worthy of enumerating, then they must already perceive the collection as a homogeneous entity at some level. In this light, the only contribution of the accumulator is to assign a quantitative value to a pre-existing percept. But how does this perception of collections itself originate?

As we have discussed, number categories piggyback on other categories. You cannot enumerate fish until you know what fish are and can group fish separately from non-fish. From a developmental standpoint, this means that numerical awareness should not be possible until at least one category can be recognized. Furthermore, subsequent number perception should emerge gradually, in one context and then another, as other categories are learned. Whether or not infants use an accumulator to enumerate sets, the necessity of non-numerical categorization is a given. And because we can assume that non-numerical categorization is taking place, there is no reason to posit a domain specific process for perceiving collections as wholes. Experience at forming and contrasting groups would be sufficient.

For example, imagine a baby playing with a pile of stuffed animals. To perceive various subsets of animals (i.e., collections), the infant would need to discover ways that the animals are similar. Extensive research suggests that adults and children discover dimensions of similarity via holistic or high-similarity comparisons. Perhaps seeing two highly similar toys in a restricted space, such as a container or one's own hands, would be enough to

induce the first comparison. As each dimension is isolated, it can serve as the basis for subsequent comparisons that may, themselves, support the discovery of additional, new dimensions (Gentner, 2003; Gentner & Medina, 1997; Gentner & Namy, 2004; Goldstone, 1996; Medin, Goldstone, & Gentner, 1993; Smith, 1989). Furthermore, the contrast between matching items and nonmatching items serves to focus attention on particular dimensions (Paik & Mix, in press). That is, when two items are not only highly similar to each other but also highly distinctive from the surrounding objects, it is maximally likely that children will compare them. Thus, we can conceptualize the development of categorization as a series of groupings, contrasts, and regroupings as more and more dimensions of similarity are discovered. Because comparisons between groupings require abstraction of shared features, the domain general process of categorization has, inherent to it, the notion of collections as undifferentiated wholes.

It is an open question whether infants engage in this type of categorization. It has been argued that even very young infants perceive object categories. But this claim is based on evidence that infants respond to category boundaries in habituation experiments (e.g., Madole & Oakes, 1999; Quinn & Eimas, 1996). For example, Quinn and Eimas showed infants a series of cat heads until looking time decreased. When shown a dog head at test, infants looked significantly longer (i.e., they dishabituated). A strong interpretation of such data is that infants formed a category of cat during habituation, compared the dog head to their remembered category at test, and rejected the dog as a member of the cat category. But this interpretation assumes that habituation–dishabituation reflects an explicit comparison process when it could instead reflect an implicit attentional process (see Cohen & Marks, 2002; Schoner & Thelen, 2001).

Furthermore, these experiments provide no evidence that infants impose such categories on their perception of complex, real world scenes. In other words, do infants look into their family living room and mentally parse the scene into cats and non-cats? There is no way to tell from existing habituation experiments, because the categories in these tasks are provided by the experimenter. Competing stimuli are stripped away so that infants need only react to the regularities presented before them. And as we have seen, there is good reason to believe that infants are well equipped to respond to perceptual regularities. This does not necessarily mean that they "have" these categories yet, or that they see the world differently because of them. Similarly, in infant number experiments, the groupings to be enumerated are bounded by the experimenter. There is usually nothing to look at except the computer screen that contains a collection of two or three pictures. In this way, it is the experimenter that completes the categorization step.

Though it is unclear whether categorization *qua* grouping is present in infancy, there are stronger indications that it has emerged by toddlerhood. Children touch objects in sequences that are consistent with explicit object grouping starting at 12 months of age (Bauer & Mandler, 1989; Sugarman,

1981). Children begin grouping similar objects around 2 years of age. Thus, we conclude that the ability to group items, and thereby view collections as wholes, does not develop simultaneously with individuation, as claimed in the domain specific accounts. Instead, colligation appears to develop somewhat later. This makes sense from a learning perspective because, to form groups, one must see individuals as similar. And to see individuals as similar, one must see individuals. The other developmental implication here is that even if infants are endowed with numerical representations, without something to enumerate (i.e., an explicit grouping perceived by the infant), there would be no reason to use them. Thus, an important challenge to the domain specific position on number learning is to show what categories, if any, infants naturally recognize and enumerate in their everyday experience.

In summary, number arises from the coordination of two ideas: (1) objects can be seen as individuals and (2) collections can be seen as wholes. Domain specific accounts assume that these notions are embodied in innate processes for representing number. However, we have argued that the same ideas can and do develop from domain general processes of object representation and categorization. There is sufficient correlational structure in objects, and sufficient sensitivity to correlational structures in infants, for these notions to emerge through experience. There is good evidence for Spelke's claim that number language plays a critical role in coordinating these notions and transforming them into an explicit sense of number (Mix et al., 2005). But at the earliest stages, these ideas may not be numerical at all – regardless of how much they may seem so to those who already understand how individuals, collections, and number are related.

How do children form small number categories?

Like most concepts, the core of number concepts consists of equivalence classes. The idea of dog is largely defined by the subset of entities in the world we call ''dogs.'' Similarly, the idea of three is largely defined by the subset of collections in the world we call ''three'' (e.g., Russell, 1919). Put another way, to know what three is means to recognize threeness in a variety of situations – to see that many otherwise disparate collections can be the same in terms of number. But what draws children's attention to number when there are so many competing properties, most of which can be analyzed at the object level, rather than the group level?

Domain specific accounts solve this problem by building in specialized processes that support number categorization in two important ways. First, they direct attention toward number. That is, domain specific processes not only enable our brains to think about numbers, but also cause them to actively seek out numerical information, much like the language acquisition device tunes children in to human language. Second, these processes provide amodal media that should facilitate numerical comparisons. Children who see three cookies on a plate and three dogs in the backyard may

not perceive these collections as equivalent. But if they represent them using identical mental tokens (whether three pointers or three gatings into an accumulator), then the likelihood of noticing this similarity should be increased. Indeed, an explicit claim of these domain specific accounts is that infants use both object tracking and accumulator representations to compare collections and judge similarity. If so, then the apparent obstacles to number categorization would be overcome largely by genetic endowment.

However, the challenges of number categorization, though unique in some ways, are not all that different from categorization in general. Because we know children form many other categories without the aid of domain specific processes, we can assume that effective domain general alternatives exist that might be recruited for use in number categorization. These include (1) abstracting dimensions of similarity by making comparisons and (2) highlighting similarity by giving shared dimensions the same name. As the following review will show, these processes not only are sufficient to explain the development of number categories in principle, but are reflected in the particular patterns that have been observed in studies of numerical and non-numerical categorization alike.

Categorization via comparisons

As discussed earlier, many studies demonstrate that people isolate new dimensions of similarity by aligning items for some other reason (Gentner & Markman, 1994; Goldstone, 1996; Kotovsky & Gentner, 1997; Markman, 1997; Smith, 1989, 1993). For example, children may not realize that dogs have tails, but if they start to examine and compare two dogs for some other reason (e.g., the way the dogs moved or sounded), they might discover that both dogs have tails, whereas people do not. Similarly, children might discover that objects in two collections can be aligned (thereby discovering numerical equivalence) because they noticed how the objects in one set can be matched one-to-one with the objects in another (e.g., cups onto saucers).

There is abundant evidence of the gradual identification and accrual of different points of alignment in non-numerical categorization. One indication is that early comparisons depend on a high degree of similarity along many dimensions – not just those relevant to a particular task (Brown & Kane, 1986; DeLoache, 1989; Gentner & Rattermann, 1991; Gentner & Toupin, 1986; Holyoak, Junn, & Billman, 1984; Smith, 1993). For example, DeLoache (1989) tested children's understanding of models by hiding a toy in either a full-size room or a model room and then having children search in the analogous space (e.g., if the toy were hidden in the room, they would search in the model, and vice versa). Children performed much better in this task when the surface similarity between the room and its model was high – that is, when the furniture had the same fabric, when the tables were the same shape and colour, and so forth – even though these features were irrelevant to the search task.

Smith (1989) reported similar effects in an object-grouping task. Using a follow-the-leader procedure, she asked children to group objects that were the same colour. For example, if she chose a red triangle and a red circle from a pile of several objects, the child was supposed to infer the commonality and produce another pair of objects in the same category (e.g., red things). Because the youngest children Smith tested could only pair items that had a high degree of similarity overall (e.g., two red circles), she concluded that children do not isolate separate dimensions of similarity at first. Instead, they initially group items with a high degree of overlap.

Additional studies also have demonstrated that exposure to high-similarity comparisons can induce children to discover new dimensions of similarity (Gentner & Markman, 1994; Gentner & Namy, 2004; Kotovsky & Gentner, 1997; Marzolf & DeLoache, 1994; Medin, Goldstone, & Gentner, 1993; Spalding & Ross, 1995; Waxman & Klibanoff, 2000). For example, Kotovsky and Gentner (1997) found that 4-year-olds had great difficulty in recognizing the relation between circles that increased in size and squares that increased in darkness. However, when children were trained on same-dimension comparisons (e.g., all sets that increased in size), their performance on cross-dimension comparisons increased significantly.

Along similar lines, Sandhofer (2003) found that 24-month-olds isolated the dimension of texture more readily when they were encouraged to compare and contrast objects. Children were trained to recognize different textures in one of two conditions. In non-comparison training, they were given three objects, one at a time, and asked, "Is this scratchy?" In comparison training, they were given the same three objects simultaneously and instructed to point to the scratchy one. Although children in both conditions learned the texture words, only children who had received comparison training could match same-textured objects in a subsequent generalization task. Thus, comparing objects seemed to support the discovery and abstraction of new dimensions of similarity.

If these processes underlie learning about number, then children's numerical equivalence judgments also should progress from high- to low-similarity matches. This should be evident in the natural progression of children's learning, as well as the effects of high-similarity training. With regard to the first point, a progression from high- to low-similarity matches is precisely what Mix and others have found in the development of numerical equivalence judgments (Huttenlocher, Jordan, & Levine, 1994; Mix, 1999a, 1999b, 2002, 2004; Mix et al., 1996; Siegel, 1971, 1974). Starting at age 3 years, children can match nearly identical sets, such as two black dots and two black disks.[1] However, children fail to match numerically equivalent sets

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1 Though the equivalent sets in this condition were matched along several non-numerical dimensions, such as color and shape, the same was true of the distracter sets, also black dots. Thus, to be correct in the high-similarity comparison, children had to take quantity into account.

where the objects are not identical, such as two black dots and two lion figurines, until 3–3.5 years of age (Mix, 1999a, 1999b, 2002; Mix et al., 1996; Sandhofer & Mix, 2003a). By 4 to 4.5 years, children can match heterogeneous sets where there are no items in common (Mix, 1999b; Siegel, 1974). Four-year-olds also recognize quite disparate numerical matches between sets of sounds and items in a visual display (Mix et al., 1996). However, number categories are not fully inclusive at 4 years of age. It takes an additional year for children to recognize numerical equivalence for dissimilar sets when one of the distracters is an identical object match (e.g., two flowers equals two trucks but not three flowers) (Mix, 2002). This condition is analogous to the cross-mapping condition that has been used in other preschool comparison research (Rattermann, Gentner, & DeLoache, 1990) and represents the complete decoupling of numerical similarity from surface or object level similarity.

These studies indicate that children do not notice numerical similarity immediately for a range of comparisons, as one might expect given an amodal representation for number. Instead, they seem to build number categories gradually, beginning with comparisons that share a high degree of non-numerical similarity, and moving over a period of years toward number in complete isolation. The length of time involved in this progression, and the particular way it unfolds, is consistent with the progressions described for non-numerical category development. This is strong evidence that the same domain general processes are at work.

Additional evidence for these processes comes from a training study with 30-month-olds (Sandhofer & Mix, 2003b). As in Sandhofer's (2003) texture training study, children were taught to identify small set sizes in one of two conditions. Children in the noncomparison condition were shown cards one at a time, and asked, for example, "Is this three or four?" Children in the comparison condition were shown three cards all at once and asked, "Which card has three?" Thus, children in the comparison condition compared sets of objects, whereas children in the non-comparison condition compared verbal labels. As for texture, children in both conditions learned the number words and could accurately identify named sets. However, only children who completed comparison training matched disparate sets in terms of numerical equivalence. This study indicates that, as with other properties like texture, children isolate and abstract the property of number by comparing collections.

But if comparison is the mechanism by which number categories are built, a key question is what makes number salient. With a range of competing cues, why would children ever notice an obscure property like number? As we will see, verbal labeling of numerical set sizes may play a major role. However, there are other potential sources of information for young children. Toddlers engage in a variety of play activities that involve implicit comparisons between sets in terms of one–one correspondence (Mix, 2002; Anderson & Mix, 2004). For example, toddlers often distribute

objects to people. This activity does not require *a priori* knowledge of numerical equivalence – the right number of objects can be achieved by making local matches (i.e., empty hand gets a toy, full hand does not). Yet the results of these efforts open a window to the idea of numerical equivalence. Toddlers also encounter a variety of objects whose functional relations with other objects encourage one-to-one mappings, such as cups and saucers, plastic eggs and egg cartons, and so forth (Mix, 2002). For example, many children play with shape puzzles, in which each piece has its own uniquely shaped hole in a wooden board. Putting the pieces in their holes is an exercise in one-to-one correspondence. Furthermore, it is selfcorrecting. The objects themselves tell you if they fit, if you have enough, or if you need more. In many instances, the holes have identical pictures of the pieces, or thematic cues that link them (e.g., a horse in the hole for the barn piece), thereby encouraging correct mappings via local pairs that have multiple points of alignable similarity. After generating enough one-to-one correspondences in situations that provide massive contextual support, children may become able to compare objects one-to-one in situations that do not (i.e., garden variety comparisons between groupings). This could be enough to support the isolation of number as a property.

Categorization via shared labels

A second domain general mechanism that promotes categorization is the use of language to name common features and relations. Like the amodal representations credited to infants in the domain specific accounts, language performs two important functions in this regard. First, shared labels signal that there is a commonality (Gentner & Rattermann, 1991; Rattermann & Gentner, 1998; Sandhofer & Smith, 1999; Waxman & Markow, 1998). Like shared surface features, a shared label can initiate comparisons that are themselves a means of discovering new dimensions. This process is reflected in the finding that objects with the same label are rated as more similar than objects with different labels (Sloutsky, Lo, & Fisher, 2001). Learning a label also facilitates the recognition of shared properties and matching (Imai, Gentner, & Uchida, 1994; Markman, 1989; Rattermann & Gentner, 1998; Sandhofer & Smith, 1999; Smith, 1993; Waxman & Hall, 1993; Waxman & Markow, 1998). For example, 21-month-olds in a triad task made more taxonomic matches (cookie–cookie) than thematic matches (cookie–Cookie Monster) if the items had been given the same nonsense label (Waxman & Hall, 1993). Labeling also has dramatic effects on preschoolers' perform-ance in cross-mapping comparisons (i.e., where relational similarity is pitted against surface similarity). For example, 3-year-olds initially failed a sticker search task in which they had to identify an item's relational match (same relative size) but ignore its identity match. However, when the experimenter labelled the items "Daddy, Mommy, Baby," 3-year-olds performed well

above chance, reaching the same levels of accuracy as children two years older (Rattermann & Gentner, 1998).

Though labels eventually do facilitate categorization, a distinctive feature of this process is a marked lag between correct labeling and correct grouping. That is, children can correctly label items along a particular dimension without seeing the items as similar. For example, Smith (1993) found that children can say that this truck is red and that ball is red, but still may not recognize that the two items belong in the same class of red things. Further tests involving a connectionist model confirmed that these two senses of "same" are quite distinct. Smith, Gasser, and Sandhofer (1997) trained a network to label three properties of a given input. For example, given a smooth red triangle and asked "What color is it?" the network learned to respond "red," and when asked "What shape is it?" the network learned to respond "triangle." However, even after learning to label objects by color the network failed to represent objects that were the same on a given property as equivalent. That is, when the network was asked, "What color is it?" and was presented with a smooth red triangle, the pattern of activation on the hidden layer was different than when the network was presented with a bumpy red square and asked, "What color is it?" The network apparently failed to isolate the property of color right away and continued to represent aspects of the shape and texture of the objects for some time even though these were irrelevant to the task at hand.

A second function of shared labels is to direct attention. Simply labeling objects in a scene improves memory for having seen the object (Gentner, 2003). This suggests that labelling increases attention toward one object in particular, and this increased processing is reflected in better memory. Shared labels also can direct attention toward a particular dimension, even if the precise meaning of the label is unknown. For example, hearing the word, "red," orients children toward the dimension of color even though they may not know exactly what "red" means (Landau & Gleitman, 1985; Backscheider & Shatz, 1993; Sandhofer & Smith, 1999). This is reflected in the fact that when 2-year-olds are asked, "What color is it?" they tend to provide a color word even though their responses are usually incorrect.

In summary, naming promotes categorization by signaling a commonality between two entities and by drawing attention toward a particular dimension. The way these processes typically unfold produces three distinct patterns: (1) labeling increases matching; (2) labels are learned prior to abstract categorization; and (3) labels direct attention toward an overall dimension before specific word meanings are learned. Let us consider next whether these same patterns are evident in the development of number categories.

Labeling increases number matching

Across several experiments, children recognized more numerical matches if they knew the labels for at least a few small set sizes (e.g., could count

to two and produce sets of one and two on demand) (Mix, 1999a, 1999b; Mix et al., 1996). In fact, children who failed to demonstrate at least this level of counting ability could not recognize numerical equivalence except for sets whose elements were nearly identical. This suggests, albeit indirectly, that knowing the labels for small collections facilitates numerical comparisons.

Crosscultural research also indicates that number words facilitate numerical comparisons. Gordon (2004) studied numerical equivalence judgments in the Pirahã, an isolated group of hunter-gatherers in the Brazilian Amazon. The Pirahã have little contact with mainstream Brazilians and are essentially monolingual. Remarkably, the Pirahã lack a true counting system. According to Gordon, the Pirahã number words correspond to ''one,'' ''two,'' and ''many'' only. Moreover, these words are inexact. For example, the word for ''one'' frequently refers to quantities of two, three, or more objects. When members of the Pirahã tribe were asked to remember the numerosity of various sets, their performance was clearly impaired, particularly for numerosities of three or more. For example, after inspecting a set of nuts for several seconds, they watched as the nuts were placed in a can and then withdrawn one at a time. After each nut was withdrawn, participants were asked whether nuts remained in the can. Even for sets as small as two nuts, the Pirahã people were only 70% correct. For sets of four, performance dropped to 40% correct. When asked to discriminate between very small quantities, such as three versus four, Pirahã performance was at chance. These findings suggest that learning words for exact quantities provides critical support for numerical reasoning.

Number labels emerge before abstract categorization

Though number naming can promote categorization, knowing number words does not result in immediate abstraction. Recall that children fail to recognize numerical equivalence between very disparate sets, even though they can accurately label small sets (e.g., Mix, 1999a, 1999b). Thus, as for other properties, children may label number in isolated instances before they know that these disparate situations are related. Sandhofer and Mix's (2003b) number training study provides additional evidence. Although 30-month-olds successfully learned the meanings of small number words via no-comparison training (inasmuch as they could identify displays of each set size when requested), they were unable to match numerically equivalent sets. The same pattern has been reported in naturalistic observations (Mix et al., 2005). In brief, toddlers accurately label small sets in restricted contexts for many months before they can match the same set sizes in experimental tasks. (See Mix et al., 2005, for details.) This protracted time course does not seem consistent with the domain specific claim that children map number words to pre-existing, amodal representations. Instead, it suggests a more gradual learning process in which children first map number words to specific

contexts, eventually juxtapose these contexts, make the necessary comparisons, and finally abstract numerical equivalence.

Number labels direct attention to the dimension of number

Finally, there is evidence that number labels direct attention toward the dimension of number before specific meanings are acquired. Wynn (1992a) showed preschoolers pairs of cards with different numbers of pictures (e.g., one fish versus four fish), and asked them to point to the card with a certain number of items (e.g., "Can you show me the card with four fish?"). By 2.5 years of age, children correctly inferred that count words greater than one referred to sets of multiples. This was evident because they pointed to the correct card as long as it was paired with a singleton. However, these children performed randomly when both cards depicted multiples.

Sandhofer and Mix (2003a) also found evidence of this pattern when they tracked children's acquisition of number language and concepts from 36 to 54 months of age. On average, children began to identify small sets correctly at 42 months of age. That is, they accurately produced sets of one, two, or three when asked. However, this accomplishment was preceded by an awareness that the number words refer to numerosity at age 37 months. In particular, children who were asked, "how many?" usually responded with a number word, even though the specific word did not always match the specific quantity. Note that both of these developments preceded the ability to match disparate sets on the basis of numerical equivalence (observed, on average, at 48 months).

How do children bring meaning to small number words?

Domain specific theorists assume, following Fodor (1983), that one can only learn words for concepts that one can already represent (Spelke & Tsivkin, 2001). From this perspective, it is natural to posit that innate representations of number provide conceptual referents for the number words. Without such representations, how else could number words be learned? And as it happens, the proposed representations make remarkably good referents. They are abstract and amodal. The category boundaries are clear, at least for small numerosities. Therefore, in these accounts, the main challenge to learning small number words is determining which words refer to which numerosities.

Once these mappings have been sorted out, children can achieve new levels of understanding. For example, Spelke (2003; Spelke & Tsivkin, 2001) has argued that mapping small number words to both the object tracking and accumulator representations for those set sizes allows children to combine the ideas of individual and collection, thereby achieving true concepts of number. In other conceptualizations, the litmus test for innate concepts is whether they appear prior to language mastery (e.g., Carey,

2001). The argument is that if children exhibit some understanding prior to mastering the words for it, then it must be innate. If they do not exhibit the understanding until after they have mastered the language for it, then it must be a cultural construction. Thus, like passing through a doorway, children move from one level of understanding to another, by way of small number word acquisition.

Though we agree that exposure to number language likely precipitates new conceptual growth, we have argued that number words do not map neatly onto pre-existing representations (Mix et al., 2005). Instead, the process is much more iterative, continuous, and interwoven than these accounts suggest. In particular, we have argued that partial understanding of number language, and even the attention-directing role of unfamiliar labels, contributes to the construction of number concepts even though neither the concepts nor the words have been mastered. Because the details of this proposal are presented elsewhere (see Mix et al., 2005) we will not reiterate them here. However, we will review four key points that are particularly relevant to the question of domain specificity: (1) Number words are part of early input; (2) Initial mappings are context-specific; (3) Number language is acquired like other language; and (4) Number word learning varies across children.

Number words are part of early input

Domain specific accounts imply that number words do not influence quantitative thought until children master them, around age 4 years. Prior to this milestone, children are thought to rely on their innate representations (i.e., object tracking and accumulator) to perform numerical tasks. Furthermore, these representations are thought to change very little, if at all, during the preverbal period. However, it is important to acknowledge that number words are part of children's input from very early in life (e.g., Durkin, Shire, Riem, Crowther, & Rutter, 1986). This means that there is a large window of time between children's first exposures to the number words and their eventual mastery of them. Within this window, it is possible that conceptual change is precipitated and shaped by exposure to partial understanding of these words.

Let's consider how this might work. Words are potent organizers of attention even when children are unsure of their meanings (e.g., Gentner, 2003). So when Mom points to two cups and says, "two," children will at least look at the cups even if they don't know what Mom is talking about. Seeing two cups that are distinct from other objects in the scene may be enough to impart the idea of "same," or the category of "cup" (Paik & Mix, in press). Indeed, there is evidence that children's early uses of "two" reflect a confusion between numerosity and similarity (Mix, 2004; Mix et al., 2005). That is, they use "two" to mean "same," but overwhelmingly do so for pairs, perhaps because pairs are easier to compare. This means that,

although children may not understand "two" at first, exposure to this word is likely directing their attention toward situations that pave the way for that understanding to develop (i.e., pairs of easily compared, high-similarity objects).

Diary and longitudinal research provides further evidence of this iterative, bootstrapping process. Correct usage of small number words develops in a stepwise progression that extends over a rather protracted time period (Mix et al., 2005; Wagner & Walters, 1982). Children first use "two" correctly in informal situations. Soon after, they begin to use "one" correctly. After several months, they begin to use "three" and "four" but are frequently incorrect. After approximately one year of correct labeling with the word "two," they start to label correctly using the word "three," but only in informal activities. Throughout this period, children provide no evidence that they comprehend any of the count words on experimental tasks. It is not until children consistently use "one," "two" and "three" with perfect discrimination in everyday situations that they begin to demonstrate correct comprehension and production of these terms in experimental tasks. Soon after, they begin labeling sets of four correctly in informal usage. At this point, nearly two years after children's initial uses of the word "two," they discover the connection between counting and cardinality (Wynn, 1990, 1992a). That is, they realize that the count "1–2–3–4" means the collection has four items in it.

Over the same time period, children's "nonverbal" number concepts also undergo significant, seemingly continuous change (Mix, Huttenlocher, & Levine, 2002b; Schaeffer, Eggleston, & Scott, 1974). As we have discussed, they accrue experience with a variety of one-to-one mappings, starting with sets that can be mapped easily via local pairings, including socially reinforced activities (distributing objects or turn-taking) and objects that invite one-to-one pairing (peg–hole, peg–hole, etc.). These activities gradually give way to set-to-set comparisons where local pairings are less obvious (car–tree, car–tree, etc.). Around the same time (3 years of age) children begin to match high-similarity, equivalent sets explicitly in experimental tasks (Mix, 1999a, 1999b; Mix et al., 1996). From there, it takes almost two years before they can match equivalent sets that are cross-mapped with object similarity (Mix, 2002). Along the way, they gradually recognize equivalence in increasingly abstract comparisons, including those between non-identical object sets, heterogeneous object sets, and sets of events and objects.

This pattern of acquisition suggests that, rather than mapping number words to pre-existing concepts, language and concepts both develop – if not hand in hand, then at least concurrently. In both cases, development is piecemeal. Learning number words involves the gradual accrual of partial understandings. So does learning number categories. This means that at any given point in time, children have an array of partial understandings at their disposal, both verbal and nonverbal, that can be assembled in

different combinations depending on the task. This suggests that words and concepts interact all the way down the line, not only at the point when children seem to understand what the words mean.

Initial mappings are context-specific

Domain specific accounts describe the mapping of number words to concepts as if it takes place at an abstract level, divorced from any particular context. This makes sense because the innate representations of number supposedly provide a context-free, amodal redescription of different set sizes. Words, like these representations, also are arbitrary symbols not tied to any particular context. If these are the components that children are mapping, then it is reasonable to think that they would do so at an abstract level.

However, children's early uses of count words do not reflect abstract mappings. Instead, they are decidedly context-specific. Mix (2004) found that when her son, Spencer, began saying number words, he mapped them to referents in a series of distinct, context-specific situations. In his earliest mappings, he did not reference sets of objects at all. Instead, he used number words to label written numerals. This began with the numerals that appeared in several of his board books, but he eventually came to recognize numerals on signs, license plates and addresses as well. At 23 months, he began using number words to label sets of objects. His first mappings were restricted to the number "two" and they always occurred within a particular linguistic frame: "Two ____. One. Two." For about a week, he labelled only sets of shoes using this frame (i.e., "Two shoes. One. Two."). Then he extended to other object sets, including two dogs, two spoons, and two straws, using the same frame. At 24 months, he began using the variant "____. ____. Two ____." For example, for two trains, he would say, "Train. Train. Two trains." This frame appeared frequently for the next 6 weeks and, during this period, he did not label sets numerically without using it.

Throughout this period, Spencer failed all tests of conventional counting. In the Give-a-Number Test, he failed to produce two objects on request and when asked how many objects were in a set of two, he responded with an idiosyncratic string of number words. Thus, although he correctly labeled different sets of two, his use of the number word "two" was far from decontextualised. In fact, it was deeply contextualized in two ways. First, it was initially restricted to specific situations – first labeling numerals, then labeling shoes. Second, these early attempts were embedded in specific linguistic frames. A similar pattern was reported in a diary study that tracked the development of another young boy (Blake) from 18 to 49 months of age (Mix et al., 2005). Blake's first number word also was "two", initially used only when asked his age (this response had been reinforced in preparation for his birthday). Although this was likely a simple association

without cardinal meaning, it is noteworthy that his first use of a number word occurred only in this situation.

Number language is acquired like other language

If the acquisition of number language is guided by skeletal principles, or supported by core knowledge, then it should develop differently from acquisition of other language. Indeed, the key claim of domain specific accounts in development is that learning in certain domains is not like other learning. However, diary and longitudinal studies indicate that children learn number words exactly the same way as they learn other words – most notably, the names of other properties. We have touched on several of these parallels already. Children realize that number words refer to numerosity before they know the specific cardinal meanings of these words (Sandhofer & Mix, 2003a; Wynn, 1992a), just as they realize that color words refer to the dimension of color before they know the meanings of individual colour words (Backscheider & Shatz, 1993; Landau & Gleitman, 1985; Sandhofer & Smith, 1999). Furthermore, as in learning color terms, local mappings of number word to referent often precede the formation of numerical equivalence classes. Both Blake and Spencer spontaneously labeled various object sets, containing the same number of items, for many months before they could match equivalent sets in a forced-choice task (Mix et al., 2005).

There are interesting parallels between the order of the mappings children perform for number and those observed for word learning more generally. As we have seen, equivalence classes are affected by the degree of similarity between objects. When the target and choice objects are highly similar, for example a red aeroplane and a similar red aeroplane, even children who do not comprehend color terms can match these objects by color (Soja, 1994). But when the target and choice objects are less similar or when there is competing similarity from a distracter object, children fail to match objects by color until long after they have learned to comprehend and produce color terms correctly (Rice, 1980; Sandhofer & Smith, 1999; Smith, 1984). This is precisely the same pattern we described previously for numerical equivalence judgments, number words, and object similarity (Mix, 1999a, 1999b, 2004; Mix et al., 1996).

A second ordering of interest involves first uses of the number words. Both Spencer and Blake mapped number words onto written numerals early in development. In fact, these constituted their first number-word mappings. This makes sense given that children tend to interpret new words in terms of shape as their vocabularies increase (Smith et al., 2002). Indeed, children with a strong shape bias can identify more letters of the alphabet than children who lack the shape bias, presumably because learning letter names requires careful attention to shape (Longfield, 2004). When children map number words to written numerals, they may be extending the shape bias to numbers. This is particularly likely given that numerically equivalent

sets do not have a consistent shape. Thus, written numerals would provide a more straightforward mapping.

Finally, Spencer's use of number frames is reminiscent of children's use of pivot grammar more generally. Bloom (1993) noted that children often use the same simple sentence structures to incorporate new vocabulary. For example, they might learn the frame "Give ____" to request items and then use this frame repeatedly as they acquire new words (e.g., "Give milk", "Give toy", "Give cookie", etc.). Spencer's number frames have much the same quality. They provided a way for him to incorporate new sets into his category of "twoness."

Thus, we see many parallels between number development and development in other domains. The significance of these parallels is that they indicate common underlying processes. When children associate number words with the dimension of number, they are likely responding to patterns in linguistic input as they do when learning other words (Bloom & Wynn, 1997). When children map number words to shape, they are using the same strategy that works in other word learning situations (e.g., Landau, Smith, & Jones, 1988; Smith, Jones, & Landau, 1992). When they overgeneralize, they are struggling to reconcile their understanding of the underlying categories with the socially accepted categories to which words refer (e.g., Mervis, 1985). These parallels provide important insights into the mechanisms by which children integrate number language with conceptual understanding – mechanisms that are sufficient to explain this accomplishment without invoking domain specific processing.

Number word learning varies across children

We have argued that number word acquisition progresses in a consistent pattern, much like acquisition of other properties. We have explained this consistency in terms of shared domain general processing. However, another way to look at this consistency is that it reflects the universality of domain specific processes. The argument is that without universal, domain specific processes built-in, the diversity, inadequacy, and pluripotentiality of children's experience with number would yield a wide range of developmental outcomes. Of course, universality is a relative thing. Domain specific theorists assume that interactions between innate structures and the environment are necessary, and therefore expect a certain amount of variability across children (Gelman, 1998). However, evidence for uniformity across children is generally taken as evidence for the constraints and focus provided by domain specific learning.

To sort these issues out, it is helpful to consider in what particular ways children differ and in what particular ways they do not. For example, we have found that although most children will reach the same endpoint in number word learning (i.e., forming number categories for small set sizes and mapping the number words to these categories), individual children

take different pathways to get there (Sandhofer & Mix, 2003a). In a longi-tudinal study of children's numerical equivalence judgments and number word acquisition, we identified two distinct patterns of acquisition. These patterns arose from children's performance on two tasks: (1) a number categorization task in which children were asked to, for example, "Give me three," and (2) an equivalence matching task in which they matched a standard to its numerical equivalent.

One group of children learned counting skills early. These children produced the correct number of objects in the give-a-number task from early on. In fact, these children demonstrated this understanding of the small number words an average of 2 months before they could match any numeri-cally equivalent sets. When these children finally began to match sets by number, they did so for all of the small set sizes nearly simultaneously. This pattern of acquisition suggests that children were mapping the number words to different instantiations without seeing these instantiations as related.

The other group of children appeared to master each number, one at a time, succeeding on the number categorization and equivalence task for the quantities of two before mastering the tasks for quantities of three. This pattern of acquisition suggests that children were fully working out the meaning of each number word, including its corresponding equivalence task, before moving on to the next. Unlike the language-first group, these children seemed to have some sense of numerical equivalence, but one that may have been contextually encapsulated until they learned the associated number words.

What might account for these differences in development? Earlier in this chapter, we outlined a variety of domain general processes that might promote numerical development. These included use of labels to cue similarity, formation of number categories via one-to-one mappings and implicit categorization, and local mappings of words to specific situations. It seems that every child would not need every mechanism to solve the prob-lems of early number development. Hence the individual differences we have described here could indicate that different children recruit different pro-cesses depending on their learning histories. For example, children who are surrounded by playthings that invite one-to-one correspondence may be more likely to form equivalence categories first. Children whose parents label sets for them often may be more likely to lead with categorization based on shared labels. A fruitful direction for future research may be to explain why some children recruit different learning mechanisms than others, based on variations in parent input, children's play environments, and preferred learning styles (e.g., object manipulation versus conversation).

What about the babies?

Thus far, we have argued that domain specific mechanisms are not needed to explain how children learn the meanings of small numbers. We have

described domain general processes that could support attention toward discrete number as well as the formation of numerical equivalence categories. We have shown that these processes not only provide plausible alternatives to domain specific processing, but seem to be reflected in the details of early numerical development. Based on parsimony alone, the evidence strongly favours a domain general account. However, we cannot be sure that nature is parsimonious. If there were additional evidence that could be explained only with recourse to domain specific mechanisms, then we would have to concede their existence. And if these processes exist, then it is reasonable to assume that they solve all the problems that domain specific theorists claim that they solve.

One class of evidence emerged in the 1980s and seemed to require a domain specific explanation of number development – namely, evidence of numerical sensitivity in infants. In short, a series of studies revealed numerical processing that was so sophisticated, in babies who were so young, that it seemed impossible to explain its existence without assuming that humans are born with a number sense (e.g., Antell & Keating, 1983; Starkey, Spelke, & Gelman, 1990; Wynn, 1992b). From there, specific representations of number were proposed (i.e., object tracking, preverbal counting, etc.) that were consistent with the particular patterns of strengths and weaknesses in infants' numerical performance. This was the genesis of current domain specific accounts.

Findings of numerical sensitivity in infants may be consistent with the domain specific accounts. However, we do not believe that they require them. First, the evidence of numerical processing in infants is not as solid as initially believed. In a recent review, Mix, Huttenlocher and Levine (2002a) concluded that none of the existing studies had succeeded in demonstrating sensitivity to number *per se*. Instead, all the procedures used with infants allowed at least one non-numerical cue to covary with number. For example, the most replicable evidence of numerical sensitivity in infants comes from looking time experiments in which even newborns have responded to changes in set size (Antell & Keating, 1983; Starkey & Cooper, 1980; Starkey, et al., 1990; Wynn, 1998; Xu & Spelke, 2000). In these experiments, infants are shown a series of displays with the same number of pictures (e.g., two dots). Over time, infants lose interest in these displays and stop looking at them as long. Once looking time has decreased by about half, infants are shown displays with a new set size (e.g., three dots). In general, infants' looking time has increased significantly in response to this change, suggesting that they remembered the first set size and noticed that the second set size was different. The problem is that when number changes in these displays, so do a variety of other cues, including contour length, density, complexity, surface area, brightness, and spatial frequency. This means that infants could be responding to a change in one or all of these non-numerical cues, rather than number.

Direct support for this interpretation came from a study in which number was pitted against contour length at test (Clearfield & Mix, 1999). Infants

were habituated to one set size of black squares. At test, they saw either the same number of squares with a different total contour length, or a different number of squares with the same contour length. Though there was significant dishabituation to the contour length change, infants did not respond to the change in number when contour length was held constant. This provided strong evidence that in previous studies where non-numerical cues were not controlled, infants were responding to changes in those cues, and not to the changes in number.

Other studies have demonstrated more sophisticated numerical abilities in infants. These include habituation–dishabituation to events or sounds (Sharon & Wynn, 1998; Wynn, 1996), violation-of-expectation experiments involving simple addition and subtraction problems (Simon, Hespos, & Rochat, 1995; Wynn, 1992b), and intermodal matching of numerically equivalent sets (Starkey et al., 1990). However, in every study, there was at least one non-numerical cue confounded with number (Mix et al., 2002a). We are unaware of any work published since that time that has succeeded in overcoming these confounds. And in several cases, when even one of these non-numerical cues has been controlled, infants have failed to respond to number (Demany, McKenzie, & Vurpillot, 1977; Feigenson, Carey, & Spelke, 2002; Lewkowicz, Dickson, & Kraebel, 2001). Thus, the very evidence that inspired domain specific theories of number development appears to reflect domain general perceptual processing instead.

But what if an infant study did show unequivocal sensitivity to number? Would that necessitate a domain specific explanation? We are not convinced. As we discussed earlier, there is strong evidence that human infants are quite sensitive to statistically reliable patterns in perceptual experience. With relatively brief exposure to correlated percepts, they not only isolate recurring patterns, but also recognize these in novel situations (Gomez & Gerken, 1999; Kirkham, et al., 2002; Saffron, et al., 1996). Most infant number experiments have been carried out with babies 5 months old or older. But even a 1-month-old has had so much exposure to objects and visual scenes, literally millions of data points (Blumberg, 2005), that it would be plausible for them to respond to changes in set size based on early learning, rather than an innate endowment. During our review of the object individuation literature, we noted that infants acquire a range of cues in a predictable sequence – one that is consistent with the gradual isolation of multiple, statistically reliable associations among correlated features. Thus, even if we assume, for the sake of argument, that infants exhibit true numerical sensitivity, there is no reason to assume that this sensitivity is built in, rather than constructed via domain general processes.

Conclusions

This chapter has focused on the key problems children must solve as they construct a concept of number. Domain specific accounts have been

developed to explain how children solve these problems. However, our review suggests that number development can be explained solely in terms of domain general processes, including those for pattern recognition, categorization, comparison, and naming. Not only do these processes provide plausible explanations for number development, but they are evident in the specific patterns obtained in the extant studies. That is, number development bears the signatures of the particular processes that have been documented in the development of other concepts and categories. The main source of evidence that seems to compel a domain specific account, numerical sensitivity in infants, is undermined by non-numerical confounds. However, at a more basic level, we question whether awareness of any property in infants requires a domain specific knowledge structure when such awareness could arise from sensitivity to statistical patterns and massive sensory input. In summary, while it is not possible to rule out domain specific processing for number conclusively, we find nothing in the evidence that compels it.

Research on early number concepts has focused on the development of domain specific accounts, and the refutation of them, for some time. Although (or perhaps because) domain general processes are assumed to exist by most investigators, there has been little interest in understanding how they contribute to numerical development. This is unfortunate because, from any perspective, these processes must play a central role. They are the blue-chip stock of developmental psychology – well-established and well-understood mechanisms of conceptual change. If there are domain specific components to number learning, then these components must interact with domain general processes. If there are not domain specific components, then domain general processes shoulder the entire explanatory burden. Either way, we hope that this chapter will lead to greater acknowledgement of the power of domain general processing and interest in its impact on early number development.

Acknowledgments

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